Dynamics of Induction Charging for Multiple Particle Agglomerations with a Thin Conducting Surface Layer

Bassem F. Nader, G. S. Peter Castle, *Fellow, IEEE*, and Kazimierz Adamiak, *Fellow, IEEE*

***Abstract*—Considerable research work has been performed on the induction charging of a single regularly shaped particle. Surface conduction was also considered to investigate the effect of a thin surface layer on the rate of charge accumulation. However, in electrostatic applications, particles are typically stacked in an arbitrary array. In the investigated model, multiple spherical particle agglomerations with a 20 Å moisture layer, finite volume conductivity, surface conductivity and permittivity have been simulated. Upon exposure to the electric field, electric shielding can occur due to the proximity of other particles. This can greatly reduce the maximum accumulated charge for a shielded particle and affect the charging time. All results have been obtained using the COMSOL commercial software, based on the Finite Element Method. The simulation results show that shielding the electric field from a given particle reduced its saturation charge significantly. The results also show that the rate of charge accumulation was mainly affected by the particles' volume and surface conductivities.**

***Index Terms*—Surface conduction, induction charging, multiple particle agglomerations, electric field, numerical simulation**

I. Introduction

ELECTRIC charging of particles is common among many electrostatic applications. Electrostatic separation, precipitation and coating are typical processes that rely on different charging techniques. However, induction charging is the preferred method for conductive or semi-conductive particles since it is easy to control.

In previously conducted investigations, analytical and simulation models were developed for induction charging of regularly and irregularly shaped particles [1] [2] [3]. The model consisted of a single conducting or semi-conducting particle, grounded and facing a charged planar electrode. In some cases, surface conduction was taken into consideration by modeling a thin surface layer having a finite surface conductivity. It was found that the actual charging time for the single particle was mainly affected by different physical properties of the particle namely size, surface roughness, volume and surface conductivities. In addition a factor of particular importance is the contact area of the particle with the ground as this affects the contact resistance.

However, in practical applications particles are typically stacked arbitrarily. Little research has been performed to investigate the effect of multiple particle agglomerations. Dascalescu *et al.* [4] evaluated the induction charge acquired by cylindrical conductive bodies and the electric force acting on them. Real conductors were simulated as cylinders with various shapes and sizes in contact with a plate electrode and exposed to a non-uniform electric field. The results showed that the proximity of other bodies resulted in an electric shielding that reduced the induction charge acquired by a given particle, and thus the resulting electric force acting on it.

The aim of this paper is to investigate the effect of electric shielding on a particle's saturation charge and actual charging time in a stacked particle arrangement. It is well known that surface conductivity can be effectively altered in many practical applications by absorption of moisture on the surface of the particles [5]. Therefore, a moisture layer was simulated as a surface conductivity in the model to investigate its effect on the dynamics of the charging process for the particles. Spherical particles are assumed to make the analysis easier. Analytically, the saturation charge of a spherical particle with radius a resting on a flat grounded electrode and exposed to a uniform electric field E_0 is equal to $6.56\pi\varepsilon_0 E_0 a^2$ coulomb [6]. For particles consisting of real conductors, it takes a finite time period to reach this saturation charge, referred to as the actual charging time constant. The actual charging time constant, τ_c, is often confused with the material relaxation time constant, τ_r, which depends only on the material bulk conductivity and permittivity ($\tau_r = \varepsilon_0\,\varepsilon_r/\sigma$), where ε_0 is the permittivity of vacuum and ε_r is the relative permittivity of the material and σ is the volume conductivity of the material [7].

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B. F. Nader was with the Dept. of Electrical and Computer Eng., University of Western Ontario, London, Ontario, Canada N6A 5B9 (e-mail: bnader2@uwo.ca).

G. S. P. Castle is with the Dept. of Electrical and Computer Eng., University of Western Ontario, London, Ontario, Canada N6A 5B9 (e-mail: pcastle@eng.uwo.ca).

K. Adamiak is with the Dept. of Electrical and Computer Eng., University of Western Ontario, London, Ontario, Canada N6A 5B9 (phone: 519-661-2111 ext. 88358; fax: 519-850-2435; e-mail: kadamiak@eng.uwo.ca).

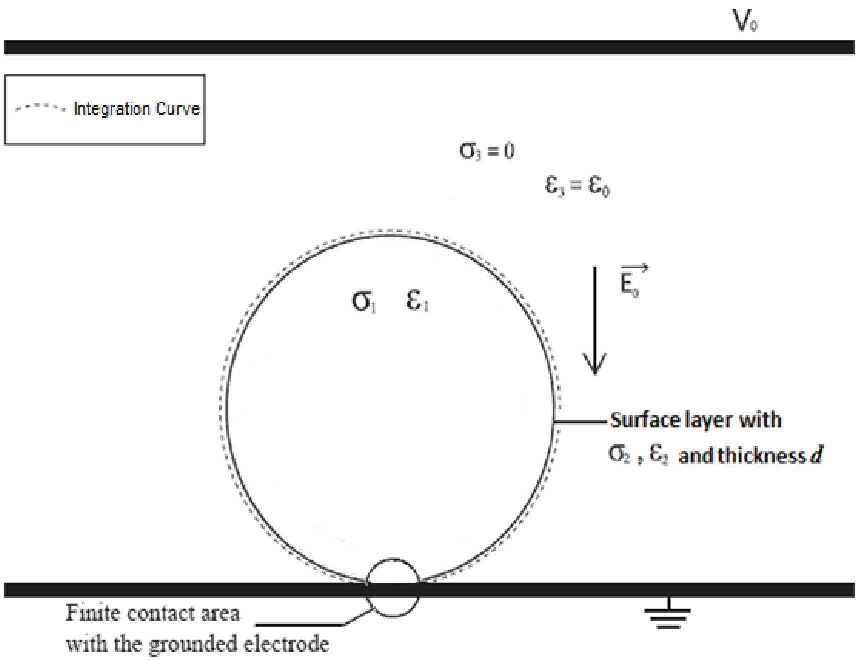

Fig. 1. Conducting spherical particle with surface layer representing moisture, resting on a grounded electrode and exposed to an external electric field.

II. Mathematical and Simulation Models

A. Mathematical Model

The mathematical model used for the single spherical particle with surface conductivity is shown in Figure 1. This geometry forms the basis for simulating multiple particle agglomerations. The particle is resting on a flat grounded electrode, facing a charged electrode of 5kV at a distance of 0.01m. Therefore, a downward uniform electric field of magnitude 0.5MV/m was produced. All spherical particles were modeled with equal radii of 1.564mm and a finite contact area with the grounded electrode of 0.039mm^2. A small finite contact area was chosen instead of a point contact. This is because a point contact in numerical models will be represented as a small, but unknown, finite contact area dependent upon the level of discretization of the domain.

The bulk material properties of the particle are defined as σ_1 and ε_1. The surface layer thickness, *d*, is chosen to be 20 Å and the conductivity and permittivity of the surface layer σ_2 and ε_2 were selected to give values of surface conductance, σ_s, reported in practical applications upon exposure of particles to moisture [5] [8]. The region separating the particle from the electrodes is defined as an air gap having σ_3 = 0 and $\varepsilon_3 = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

According to Gauss' law, the total charge accumulated on the spherical particle will be equal to the integral of the normal electric displacement D_n at the surface:

$$Q_s = \iint \rho_s ds = \iint D_n ds \quad (1)$$

where ρ_s is the surface charge density, D_n the normal component of electric displacement and Q_s the total saturation charge.

The actual charging time constant can be evaluated from the charging process of the particle described in terms of the time it takes for the charge to reach its saturation value. This is directly affected by the volume and surface conductivities. The charge as a function of time can be expressed as [7]:

$$Q(t) = Q_s(1 - \exp(-t/\tau_c)) \quad (2)$$

where Q_s is the saturation charge of the material and τ_c the actual charging time constant. For $t = \tau_c$ equation (2) reduces to:

$$Q(\tau_c) = Q_s(1 - 1/e) = 0.632 Q_s \quad (3)$$

Therefore, the time after which the particle accumulates 63.2% of its saturation charge is defined as the actual charging time constant.

In electrostatic applications, surface conduction is important as it directly affects the charging dynamics of a given material. In general, for a volume with a surface layer, surface conductivity can be defined using the equation of a resistance as:

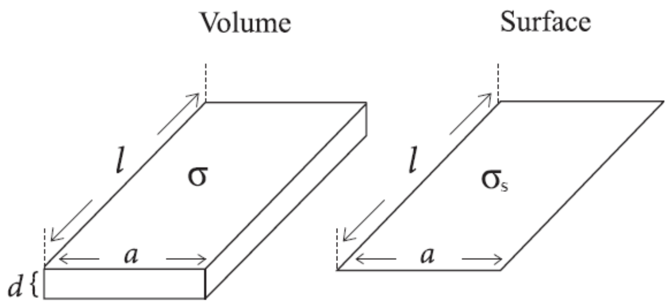

Fig. 2. Volume and surface conductivities for the material and surface layer.

$$R = \frac{l}{\sigma a d} = \frac{l}{\sigma_s} a \quad (4)$$

where R is the resistance of the material, l is the length of the layer, a the width of the layer, d the thickness of the layer, σ volume conductivity of the material and σ_s surface conductivity of the surface layer as shown in Figure 2.

Based on equation (4) the equation for surface conductance is:

$$\sigma_s = \sigma d \quad (5)$$

B. *Simulation Model*

The simulation model was created using the COMSOL commercial software. Two electrodes were represented by horizontal rectangles of 0.05m square and a separation of 0.01m. The lower electrode was grounded, while the upper electrode was supplied with 5 kV, which generates the electric field magnitude of 0.5 MV/m. The vertical sides of the rectangle were set as electric insulators.

Initially, a single spherical particle with 1.564mm radius was modeled. The surface layer for the particle was represented in COMSOL as an "electric shielding". To integrate the surface charge density and calculate the total charge magnitude from (1), an artificial spherical surface as shown in Figure 1, concentric with the particle, was created.

The model was then extended to simulate multiple particle agglomerations in different patterns. Surface layer thickness was set constant at 20 Å; all particles were identical and made point contact with each other. The contact area of the particles with the ground is constant, and equal to 0.039mm² in all cases. The simulations were conducted in the transient mode analysis to study the dynamics of the charging process. The total charge accumulated by the particle in question and actual charging time constant were calculated for the different simulations.

III. Simulation Results and Analysis

A. *Single Spherical Particle and Model Verification*

The simulations were first conducted for a single spherical particle with a radius of 1.564mm, and surface layer thickness of 20 Å. The volume conductivity of the surface layer, σ_2, was set to 0.005S/m, which is between the tap water conductivity of 0.01 S/m and fresh water conductivity of 0.001 S/m. The surface conductivity σ_s representing the moisture layer was then calculated from (5) to be 10^{-11} S/□. The particle's relative permittivity was set to ε_r= 3and the volume conductivity σ_1 = 0.1nS/m. In all cases of further simulations, these parameters were fixed.

TABLE I

Comparison between the saturation charge from analytical and numerical solutions

Particle Shape	Felici's Formulae [6]	Induction Charge |Q|(pC) (Felici)	Induction Charge |Q|(pC) (COMSOL)	Percentage Difference (%)
Spherical particle point contact with grounded electrode	$6.56\pi\varepsilon_0 E_0 a^2$	223.17	223.84	0.3%
Spherical Particle having 0.039mm² contact area	$6.56\pi\varepsilon_0 E_0 a^2$	223.17	223.51	0.15%

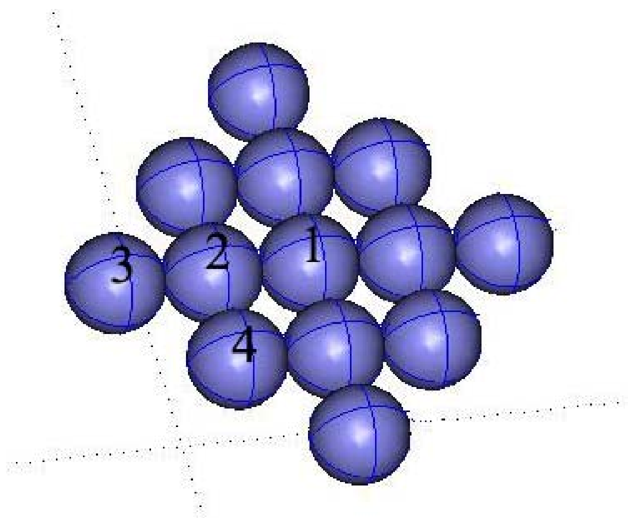

Fig. 3. Thirteen-particle agglomeration.

Two cases of simulation were performed for the single particle. The first case is a particle having a point contact with the ground and was used to compare the saturation charge with Felici's analytical formula [6] and verify the model. The second case assumed the particle had a small finite contact area of 0.039mm^2 with the grounded electrode the situation assumed in the further simulations. The results are summarized in Table I.

For both cases of simulation, the percentage error between numerical and analytical results was small with a maximum of 0.3% for a particle with point contact. For a larger contact area, the maximum attainable charge is smaller, as the surface integration area for the given particle is now smaller. However, as described earlier, a small finite contact area is preferred in simulation models when the dynamic charging is considered. In this case a 0.039mm^2 contact area was sufficient to get accurate results. The accuracy of the model is therefore verified, and multiple particle agglomerations can be simulated.

B. Agglomeration of Thirteen Particles

The geometric representation of a thirteen-particle agglomeration is shown in Figure 3. All particles are placed flat on the ground electrode with a symmetrical arrangement, so that only four particles (numbered as 1, 2, 3 and 4) represent distinct cases. The charging dynamics were investigated for particles 1-4, and the results summarized in Figure 4.

The results show that Particle 1, located at the center of the pattern, had a saturation charge Q_s = -97.07pC, the smallest among the surrounding particles. The shielding of the particle is clearly shown geometrically in Figure 3. The saturation charges calculated for the other particles are larger when the particle is closer to the edge of the array. Particle 3 surrounded from only one side was found to have the highest saturation charge of Q_s = -150.08pC. When compared to a stand-alone particle, it can be seen that particles in contact with each other have a reduced saturation charge due to shielding of the electric field.

The actual charging time constants of all four particles in the pattern slightly increase with reduced shielding. For Particles 1, 2, 3, 4 and the stand-alone particle, the actual charging time constants were 12.5ms, 12.7ms, 18.3ms, 16.3ms and 20ms, respectively. Particle 3 with the weakest shielding effect has the slowest accumulation of charge amongst the shielded particles. This is a surprising effect, as the less shielded particles have a higher saturation charge, so it would be intuitively expected that the charging time constant would be higher. The results have also shown that $\tau_c \ll \tau_r$ where

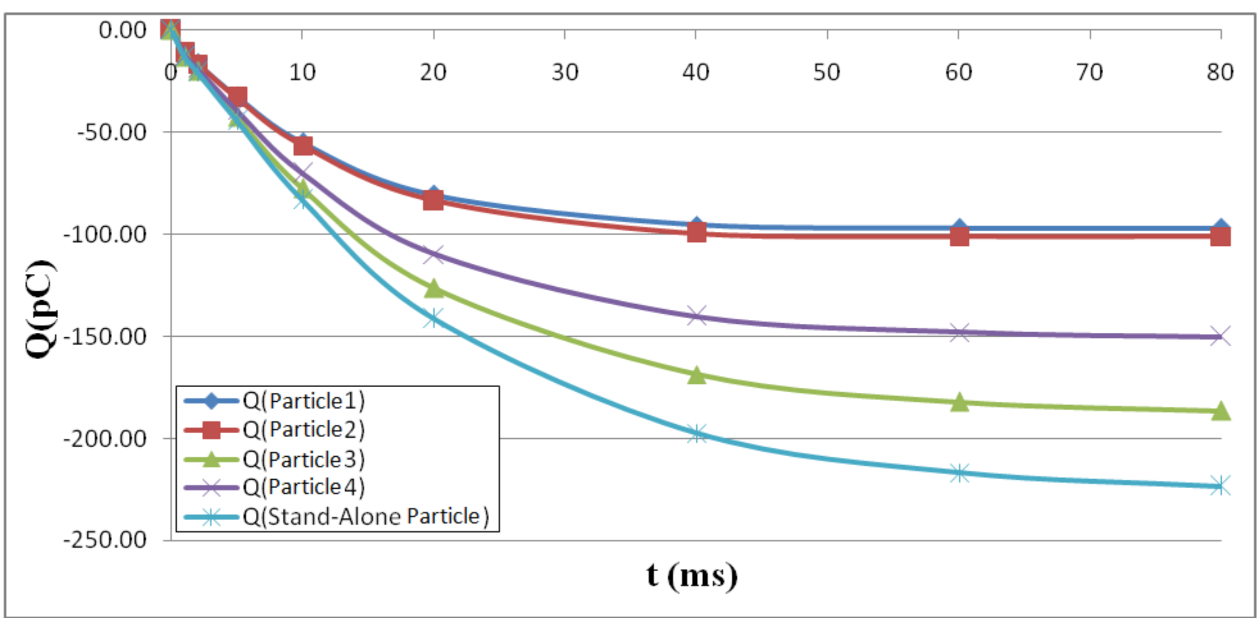

Fig. 4. Induction charge Q versus time t for the particles 1-4 compared with stand-alone particle.

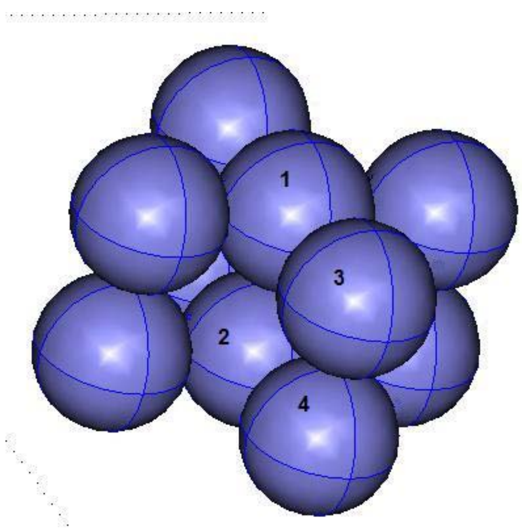

Fig. 5. Five-over-five particle pattern.

τ_r = 0.266s. This is because τ_r depends only on the bulk conductivity and permittivity, whereas the actual charging time is also affected by the surface conductivity. Therefore, actual charging time constant and saturation charge were found to be directly affected by proximity of other particles. Further geometric models were also investigated for different particle patterns to investigate the electric shielding effect on the charging dynamics of the particles.

C. *Five-over-five Particle Pattern*

The geometry of a five-over-five particle pattern is shown in Figure 5. The results are presented for the particles 1, 2, 3 and 4 only, due to the symmetric arrangement in the pattern. The results are summarized in Figures 6 and 7.

The results show that Particle 3 which is the least shielded by other particles reaches the highest saturation charge Q_s = -310.98pC. The actual charging time constant τ_c was also the largest for Particle 3 (53.6ms) compared to Particle 1 (42.4ms). It can also be noted that Particle 3 has a saturation charge Q_s (-310.98pC) greater than that of a stand-alone particle (-223.51pC). This was due to elevation of Particles 1 and 3, which would be exposed to a stronger electric field due to less shielding from the grounded electrode.

Particles 2 and 4, shielded from the top by Particles 1 and 3, are exposed to a smaller electric field. From Figure 7 it can be seen that the charge accumulates in the beginning then decays. As the current initially flows from the ground plate to the lower particle, the charge builds up. The electric field directed downward from the upper electrode is mainly blocked, shielding Particles 2 and 4. The shielded particles are still exposed to some electric field, and this is shown as they accumulate some charge. The loss of charge noticed in Figure 6, is the result of charges moving from the lower shielded particles to the upper residing ones, i.e. Particles 2 and 4 lose charge in favor of 1 and 3. Moreover, Particle 2, shielded from all sides, is seen to lose charge almost entirely upon saturation.

It can be concluded, that the particles in the top layer acquire more charge than those in contact with the grounded electrode, because of exposure to a stronger electric field. Particles in the lower layer experience much weaker electric field, because they are shielded by the electric charge accumulated in the top layer. In the first phase, the charge increases rather quickly, because the upper layer is not yet charged. However, the charge accumulation in the upper layer decreases the electric field in the area below, reducing the electric charge of Particles 2 and 4.

D. *Increased Electric Shielding of the Central Particle*

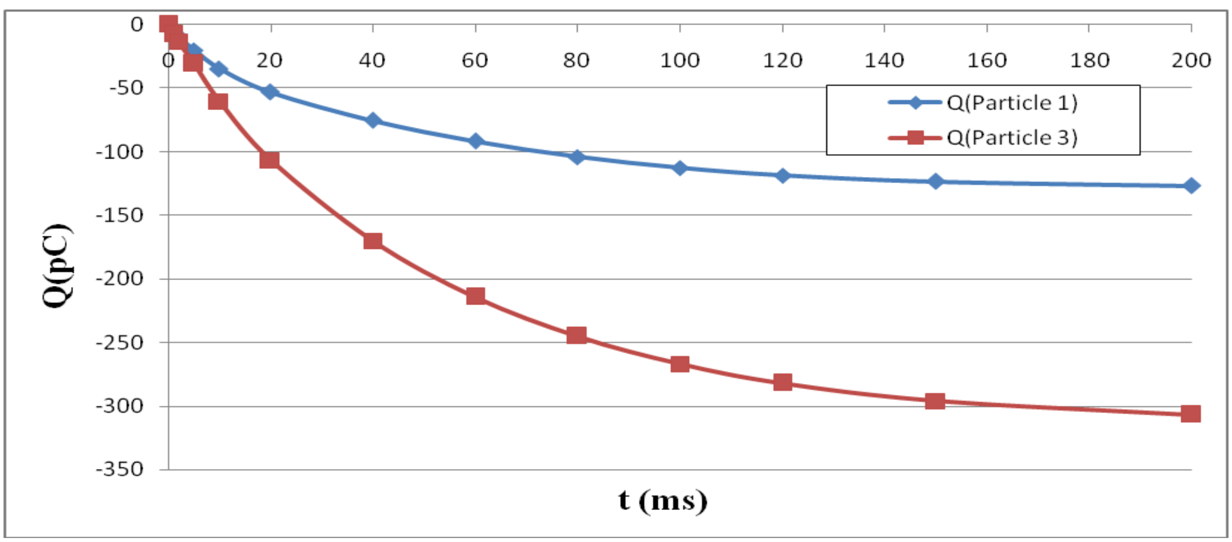

Fig. 6. Induction charge *Q* versus time *t* for particles 1 and 3.

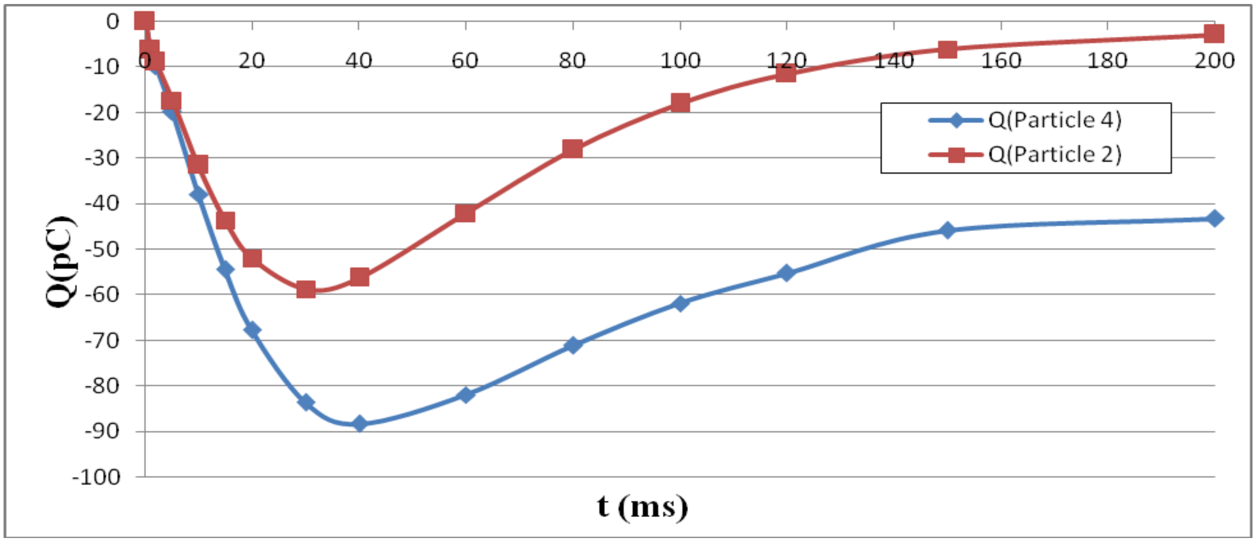

Fig. 7. Induction charge *Q* versus time *t* for particles 2 and 4.

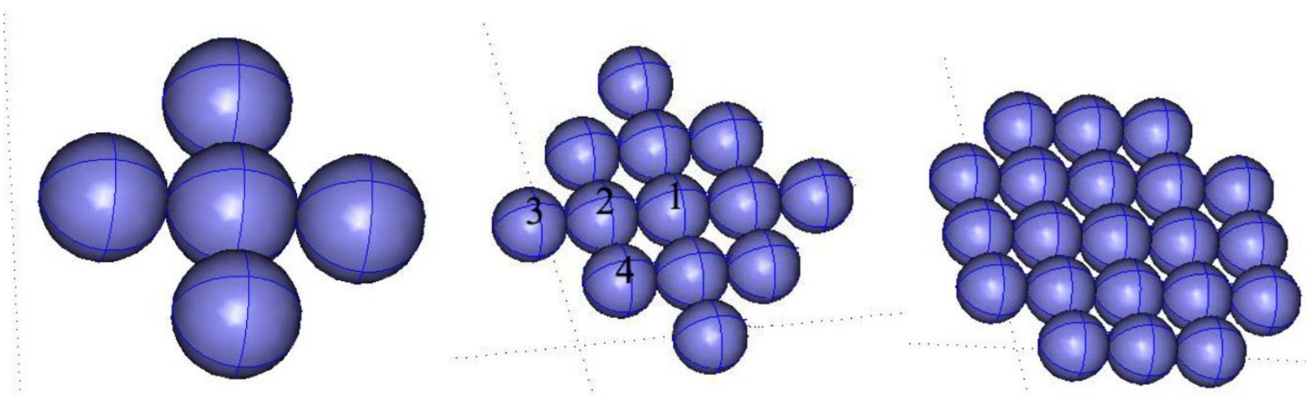

Fig. 8. (a) Five - Particle Agglomeration, (b) Thirteen - Particle Agglomeration, (c) Twenty one – Particle Agglomeration.

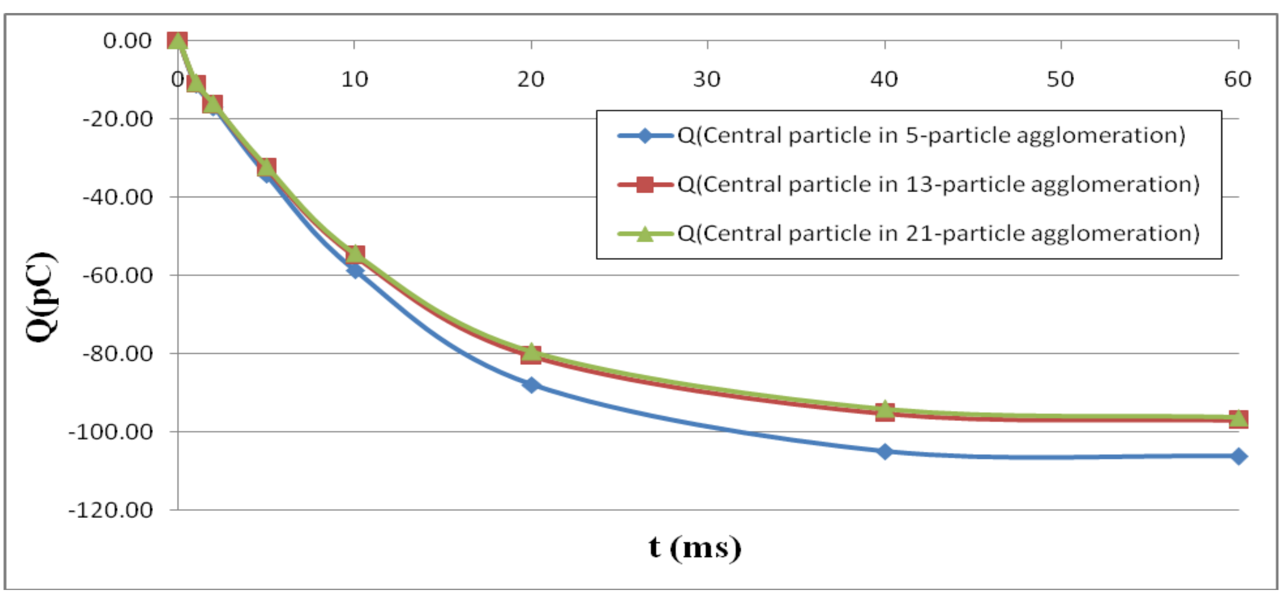

Fig. 9. Induction charge *Q* versus time *t* for the central particles in different particle patterns.

To further investigate the effect of shielding a spherical particle, flat arrangements of multiple particle agglomerations were modeled, and the results were compared for the central particle, which will be the most shielded. Figures 8(a), (b) and (c) show three different geometric models that were investigated.

The saturation charge of the central particle in the five-particle agglomeration model was the highest and equals to 106.22pC. As the number of the surrounding particles increases, the saturation charge of the central particle decreases. In the first model, four particles surround the centered sphere and it was noted that the central particle had the highest saturation charge and a slightly larger actual charging time constant of 12.8ms when compared to 12.5ms and 12.4ms for the 13-particle and 21-particle patterns respectively.

This also shows that increasing the number of surrounding particles from twelve (13-particle agglomeration) to twenty (21-particle agglomeration) does not affect the saturation charge of the central particle. The increase in saturation charge Q_s and charging time constant τ_c of the central particle changes only slightly. Therefore, it can be concluded that a particle surrounded by other particles will have a lower saturation charge and actual charging time constant. This occurs up to a certain point after which increased shielding (adding more surrounding particles) around the central particle will have no more effect on its saturation charge and rate of charging.

IV. Conclusions

In all the cases simulated, electric shielding of a particle due to proximity of other particles in a certain pattern has been shown to greatly reduce its maximum attainable charge. The results can be summarized as follows:

In the thirteen-particle agglomeration, particles that are closely surrounded by other particles experience more shielding of the electric field. The results showed a decreased saturation charge for the shielded particles. In the five-over-five particle pattern, the elevated particles showed a high saturation charge and lower shielded particles initially received charge that they subsequently lost in favor of the upper particles. Shielding the particle almost entirely has been shown to block most of the particle's exposure to the electric field, and therefore accumulate the least charge.

In all cases of simulation, upon increasing the shielding of the electric field, the actual charging time for a given shielded particle is faster. This is noticed as the particle's maximum attainable charge is less due to increased shielding of the electric field and therefore less time is needed to reach saturation.

When the number of particles was increased from five to thirteen, a significant decrease in saturation charge was observed for the central particle in the pattern. The number of particles in agglomeration was then increased from thirteen to twenty-one particles; a negligible decrease in saturation charge for the central particle was noticed upon the second increase. It was concluded that shielding the electric field by adding more surrounding particles would decrease the central particle's saturation charge up to a point where adding more surrounding particles would have no further effect on its charging dynamics or maximum induced charge.

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