Mathematical Simulation Study of Digital Signal Processing of the ESPART Analyzer for the Nanoparticle Size Range

Jeremy Stark, Jing Zhang, Rajesh Sharma, Malay Mazumder

Abstract— ESPART (Electronic Single Particle Relaxation Time) analyzer is an instrument that has been used in many applications to determine both size and charge of micrometer sized particles in an aerosol flow in real time. By applying an electric field onto particles passing through a Laser Doppler Velocimeter (LDV) apparatus and measuring relative motion of the particle in the electric field, particle aerodynamic diameter and charge can be calculated. The current development of the ESPART analyzer is in measuring nanoparticles, which requires measuring signals with a low SNR (Signal-to-Noise Ratio). The purpose of this study is to develop a simulation model and to investigate the effects of the algorithm of digital signal processing, low signal strength, and noise in the signal detection circuits on the measurement resolution of an ESPART analyzer. A general purpose model was developed for an ESPART analyzer in Matlab/Simulink. Simulation was performed to investigate the resolution over nanoparticle size ranges for signals with different signal/noise ratios and different algorithms of digital signal processing. Results were compared for accuracy and percent error of the system.

I. INTRODUCTION

Mathematical modeling and simulation can be a powerful tool for determining the viability of a proposed modification to a system. By modifying the ESPART particle analyzer to work in the nanoparticle size range, a large number of applications can be applied to this instrument that is capable of simultaneously measuring particle size and electrostatic charge [4]. Through the use of current ESPART technology the device is able to measure in the micrometer particle range. This device has been used in printing industry applications to measure size and charge distributions for copier toner [2,10], for space applications in measuring dust particles size and charge[5,6,7,8,12], and studies have shown the device can be used to distinguish between different types of bacteria[11,13,14].

In developing a new version of the ESPART particle analyzer, modifications to the signal processing system will be made, and the system can be modeled first using Matlab/Simulink. By developing a digital signal processing model, limitations of the system can be evaluated without first building a physical system. Changes can be made easily in constraints of the signal processing system, and variables of the instrument can be changed over large values to test for viability in signal processing and system performance. This work is being accomplished with the help of the National Science Foundation.

II. THEORY

The ESPART particle analyzer consists of two major systems, Laser Doppler Velocimeter (LDV) and particle relaxation apparatus, and the signal processing systems. The LDV system is a process used to determine the velocity of a particle passing through the intersection of two coherent light sources, such as lasers. Particles are introduced into the intersection of the laser beams, called a sensing volume, from above and fall perpendicularly through the plane of intersection of the beams[1,3,9]. An LDV system can give the velocity of the particles flowing through the sensing volume. By combining this process with an external exciting field, particles can be oscillated in the sensing volume with a given frequency. This exciting field can be an AC electric field, which is used to measure particle charge and size, or an acoustic field, which can measure only particle size. The particles excited by this field will experience a lag with reference to the exciting signal. This lag with respect to the exciting signal can be seen as a phase lag and shown through in (1), where $\omega_s$ is the angular frequency of the exciting signal, and $\tau_p$ is the relaxation time of the particle motion.

$$\phi = \tan^{-1}(\omega_s \tau_p)$$

After measuring the phase lag, the particle relaxation time is used to calculate the aerodynamic diameter in (2), where $\eta$ is the coefficient of viscosity of the medium, $\rho_0$ is the unit density of the flow, and $C_e$ is the Cunningham Slip Correction.
Particle charge is determined by the amplitude of oscillation of the particle in the applied AC electric field. The magnitude of oscillation compared with the magnitude of the applied electric field gives the net charge as shown in (3), where \( q \) is the net charge of the particle, \( V_p \) is the amplitude of velocity of the particle, and \( E_0 \) is the amplitude of the AC electric field.

\[
q = \frac{V_p}{E_0} \frac{3\pi \eta d_a}{C_c} \sqrt{1 + \omega_a^2 \tau_p^2}
\]  

(3)

III. SIMULATION MODEL

The signal processing simulation model for the ESPART analyzer is being developed using MATLAB/Simulink. Input to the model is particle size and charge and output is the values derived through simulation of the signal processing. Values can be compared for various exciting frequency settings to test the resolution of the system over different particle diameters.

The signal detected by the ESPART Analyzer is a Doppler burst signal, can be represented by (4).

\[
s_d = A(t) \cos(2\pi f_c t + M(t) + \phi)
\]  

(4)

Where \( A(t) \) is the magnitude of the signal from the LDV, \( f_c \) is the carrier frequency that is created by the LDV, \( \phi \) is the general phase shift, and \( M(t) \) is the frequency modulated signal as shown in (5).

\[
M(t) = \frac{V_p}{d_f} \sin(2\pi f_c t - \phi_p)
\]  

(5)

In (5), \( d_f \) is the fringe spacing of the LDV system, \( V_p \) is the horizontal velocity component of the particle due to an external AC electric or acoustic exciting signal, \( f_c \) is the frequency of the exciting signal which drives particles to move horizontally through the sensing volume, and is the phase lag of the particle related to the exciting signal.

We assume the distribution of light in the sensing volume is a normal distribution. We reproduce a Doppler burst signal in (4), in which \( A(t) \) is a Gaussian function. The reproduced Doppler burst is shown in Fig. 1.

\[
s_I = \frac{A_d}{2} \left[ \cos(2\pi f_c t + M(t)) + \cos(M(t)) \right]
\]  

(6)

\[
s_Q = \frac{A_d}{2} \left[ \sin(2\pi f_c t + M(t)) + \sin(M(t)) \right]
\]  

(7)

Taking the value of the modulated function of (8) and (9) gives us the modulated signal in (10), then taking the derivative gives us the velocity component of the Doppler signal in (11).

\[
m(t) = \frac{1}{2\pi} \frac{dM(t)}{dt} = \frac{V_p}{d_f} \cos(2\pi f_c t - \phi_p)
\]  

(11)
After filtering (12) and (13), signals (14) and (15) are left as the values of the sinusoid with frequency components removed.

\[ M_I = \frac{1}{2} V_p \cos(\phi_p) \]  
\[ M_Q = \frac{1}{2} V_p \sin(\phi_p) \]  
\[ V_p = 2M \sqrt{M_I^2 + M_Q^2} \]  
\[ \phi_p = \arctan \frac{M_Q}{M_I} \]

IV. SIMULATION RESULTS

The system model is constructed so that single trials are run for various consecutive particle sizes at a set exciting frequency to test for resolution of small phase angle and low signal strength. Trials are repeated for various exciting frequencies to obtain satisfactory data of particle characterizations for a range of particle sizes. Data from trials at 1 kHz and 100 kHz are shown in Fig. 3 and Fig. 4.

As seen from graphs in Fig. 3 and Fig. 4, resolution of particle size relies heavily on the frequency of the exciting signal, which causes the particle to experience oscillatory motion. While the exciting frequency increases, smaller particles experience motion at high frequencies with greater changes. This can be seen from the flattening curve in the 1 kHz graph, showing very small phase lag at the micrometer range. With 100 kHz exciting frequency, we see the curve flattening much lower, below the micrometer particle range. It can be inferred that a higher frequency exciting signal can be used to resolve smaller particle sizes at limits of the light
collection methods and devices such as silicon photodiodes and photomultiplier tubes. Future testing will include random noise to test signal to noise ratio.

V. CONCLUSIONS

A general purpose model in Matlab/Simulink was created for the development of a ESPART analyzer to measure nanoparticles. With the help of this simulation model it is possible to investigate the resolution over nanoparticle size ranges for signals with different signal/noise ratios and different algorithms of digital signal processing.

REFERENCES