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Electrostatic Force on a Charged Dielectric Sphere near a Grounded Conducting Plate

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Abstract

The electrostatic force on charged particles is affected by nearby conducting objects. The charge distribution on the particle is influenced by the nearby objects through induced, polarization charge. Presented here is a computationally efficient method to find the induced charge distribution on the particle. Once the charge distribution on the particle is known, the force on the particle can be calculated. The force is computed on a charged, dielectric sphere near a grounded conducting plane.

The method is based on a multipolar expansion of the particle charge distribution first given by Fowlkes and Robinson [1]. The first n terms in the multipolar expansion can be represented as an n-vector. The principle result is a matrix equation for the multipolar expansion:

$$\left(\overline{\overline{\mathbf{G}}}\,\overline{\overline{\mathbf{V}}}\,\overline{\overline{\mathbf{B}}} - \bar{\bar{\mathbf{I}}}\right)\overline{\mathbf{P}} + \overline{\overline{\mathbf{G}}}\,\overline{\mathbf{\Phi}}_{\mathrm{EXT}} + \overline{\mathbf{Q}}_{\mathrm{NET}} = 0 \qquad (1)$$

where

$\overline{\mathbf{P}}$ unknown multipolar expansion of the particle charge distribution
$\overline{\overline{\mathbf{G}}}$ known geometry matrix defining the induced charge, which depending on the particle shape
$\overline{\overline{\mathbf{V}}}$ known electric potential of a given multipolar distribution
$\overline{\overline{\mathbf{B}}}$ known matrix representing image charges in the nearby plate
$\bar{\bar{\mathbf{I}}}$ identity matrix
$\overline{\mathbf{\Phi}}_{\mathrm{EXT}}$ potential of the externally applied electric field, and
$\overline{\mathbf{Q}}_{\mathrm{NET}}$ is the uniform, net charge on the particle

Using this formulation, the force on a charged, dielectric sphere near a grounded plate is solved using Matlab and the results are consistent with the earlier work of Fowlkes and Robinson [1]. The computational efficiency is greatly improved. The number of terms in the multipolar expansion is determined by the desired accuracy. For the case of a charged, dielectric particle touching a conducting surface, several hundred terms are needed.

[1] W. Y. Fowlkes and K. Robinson, "The Electrostatic Force on a Dielectric Sphere Resting on a Conducting Substrate," Particles on Surfaces, Ed. K. L. Mittal, Plenum Publishing Corp., pg. 143 - 155, 1988.